THE MAHARAJA SAYAJIRAO UNIVERSITY OF BARODA, VADODARA

Ph. D. ENTRANCE TEST (PET) 2025

Signature of Invigilator

Roll. No. | | | | | | |

Paper - II
Statistics

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Maximum Marks: 50 **No. Of Printed Pages: 8**

Instruction for the Candidate:

1. This paper consists of **FIFTY (50)** multiple choice type questions. Each Question carries **ONE (1)** mark.
2. There is no Negative Marking for Wrong Answer.
3. A separate OMR Answer Sheet has been provided to answer questions. Your answers will be evaluated based on your response in the OMR Sheet only. No credit will be given for any answering made in question booklet.
4. Defective question booklet or OMR if noticed may immediately replace by the concerned invigilator.
5. Write roll number, subject code, booklet type, category and other information correctly in the OMR Sheet else your OMR Sheet will not be evaluated by machine.
6. Select most appropriate answer to the question and darken appropriate oval on the OMR answer sheet, with black / blue ball pen only. DO NOT USE PENCIL for darkening. In case of over writing on any answer, the same will be treated as invalid. Each question has exactly one correct answer and has four alternative responses (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example:Ⓐ ● Ⓒ Ⓓ where (B) is correct response.
7. Rough Work is to be done in the end of this booklet.
8. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
9. Calculators, Log tables any other calculating devices, mobiles, slide rule, text manuals etc are NOT allowed in the examination hall. If any of above is seized from the candidates during examination time; he/ she will be immediately debarred from the examination and corresponding disciplinary action will be initiated by the Center Supervisor as deemed fit.
10. DO NOT FOLD or TEAR OMR Answer sheet as machine will not be able to recognize torn or folded OMR Answer sheet.
11. **You have to return the OMR Answer Sheet to the invigilator at the end of the examination compulsorily** and must not carry it with you outside the Examination Hall. You are however, allowed to carry original question booklet on conclusion of examination.

Paper - II
Statistics

Note: This paper contains **FIFTY (50)** multiple-choice questions. Each Question carries **ONE (1)** mark.

01) Which of the following statements is true ?
   A) A field is closed under finite unions
   B) A class closed under complementations and finite unions can not be a field
   C) A class that contains only two events $\{\emptyset, \Omega\}$ can not be a field
   D) Intersection of two fields cannot be a field.

02) Sequence { 1, -1/2, 1/3, -1/4, . . . }
   A) is an oscillating sequence
   B) diverges to -∞
   C) converges to 0
   D) converges to 1

03) If for a real-valued function f, derivative at some point $c \in R$ exists, then
   A) f takes maximum value at c
   B) f takes minimum value at c
   C) f is continuous at c
   D) f will have discontinuity at c

04) Which of the following is called the Cauchy criterion of convergence for sequence of functions $\{f_n(x)\}$?
   A) For given $\varepsilon > 0, \exists\, N \in I\ s.t.\ n \geq N \Rightarrow |f_n(x) - f(x)| < \varepsilon$
   B) For given $\varepsilon > 0, \exists\, N \in I\ s.t.\ m,n \geq N \Rightarrow |f_n(x) - f_m(x)| < \varepsilon$
   C) Either (a) or (b)
   D) Neither (a) nor (b).

05) A series of real numbers $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$
   A) Converges absolutely
   B) Converges conditionally
   C) Diverges
   D) Oscillates

06) Co-factor of **4** in the determinant $\begin{vmatrix} 1 & 2 & -3 \\ 4 & 5 & 0 \\ 2 & 0 & 1 \end{vmatrix}$ is equal to
   A) -2
   B) 2
   C) -5
   D) 5

07) If $\boldsymbol{I}$ is an identity matrix of order $\boldsymbol{n}$, then its rank is equal to
   A) 1
   B) less than $\boldsymbol{n}$
   C) $\boldsymbol{n}$
   D) greater than $\boldsymbol{n}$

08) A system of $\boldsymbol{m}$ non-homogeneous linear equations $AX = B$ in $\boldsymbol{n}$ unknowns is consistent if
   A) $m = n$
   B) $m \neq n$
   C) $Rank\ A \neq Rank\ [A, B]$
   D) $Rank\ A = Rank\ [A, B]$

09) Inverse of an orthogonal matrix is
   A) A null matrix
   B) An identity matrix
   C) An orthogonal matrix
   D) An unitary matrix

10) The scalar λ is a characteristic root of the matrix A if
   A) $(A - \lambda I)$ is non-singular
   B) $(A - \lambda I)$ is singular
   C) A is non-singular
   D) A is singular

11) There are 10 patients in the children ward of a hospital who are monitored by 2 staff members at a time. If the probability of a patient requiring emergency attention at any given point of time by a staff member is 0.3, then assuming the need for emergency attention for the patients is independent of each other, what is the probability at any given time that there will not be sufficient staff to attend all the emergencies?
   A) 0.3828
   B) 0.3
   C) 0.09
   D) 0.6172

12) Let X be a random variable defined as number of dots on the roll of a balanced die. Then expected value of $g(x) = 2X^2 + 1$ will be equal to
   A) $\frac{47}{2}$
   B) $\frac{94}{3}$
   C) 47
   D) 94

13) Suppose a random variable X has the following probability distribution:
$$f(x) = \frac{1}{8}\binom{3}{x}, x = 0, 1, 2, 3$$
Then the moment generating function of X will be
   A) $\frac{1}{8}(1 + e^t)^3$
   B) $\frac{1}{4}e^t$
   C) $(1 + e^t)^2$
   D) e^t

14) Let 'a' and 'b' be any two real constants such that $a \le b$ and $F(x)$ be the cumulative distribution function of a random variable X. Which of the following is not true?
A) $P(a < X \le b) = F(b) - F(a) + P(X = b)$
B) $P(a \le X < b) = F(b) - F(a) + P(X = a) - P(X = b)$
C) $P(a \le X \le b) = F(b) - F(a) + P(X = a)$
D) $P(a < X < b) = F(b) - F(a) - P(X = b)$

15) Envelope I contains 2 black and 3 red sheets, while envelope II contains 3 black and 4 red sheets. From among these two envelopes, one is selected at random. The probability of choosing envelope I is double that of envelope II. If a red sheet is drawn from the selected envelope, then the probability that it has come from envelope II is:
A) $\frac{11}{31}$
B) $\frac{13}{31}$
C) $\frac{15}{31}$
D) $\frac{10}{31}$

16) Suppose two-wheelers arrive at a petrol pump as per Poisson process. If two-wheelers are arriving at the average rate of 2 per minute, what is the probability that no two-wheelers arrive in the first 5 minutes after opening the petrol pump.
A) e^{-5}
B) $e^{-5/2}$
C) e^{-10}
D) $e^{-2/5}$

17) Which of the following distributions have closed form expression for quantile function?
A) Normal
B) Exponential
C) Gamma
D) Beta

18) Suppose a p-variate normal vector $\underline{X}$ is partitioned as $\begin{pmatrix} \underline{X}_{(1)} \\ \underline{X}_{(2)} \end{pmatrix}$ with usual notations for the corresponding partitions of Σ. Then the conditional variance of $X_{(2)}$ given $X_{(1)}$ is

A) $\Sigma_{22} + \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}$
B) $\Sigma_{22} - \Sigma_{21}\Sigma_{11}^{-1}\Sigma_{12}$
C) $\Sigma_{11} + \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$
D) $\Sigma_{11} - \Sigma_{12}\Sigma_{22}^{-1}\Sigma_{21}$

19) Which of the following statements are true?
S1: When all random variables in a random vector are continuous, their joint distribution can be represented by a joint pdf.

S2: When all random variables in a random vector are discrete, their joint distribution can be represented by a joint pmf.

A) Only S1
B) Only S2
C) Both S1 and S2
D) None of the two

20) A CLT is a statement of convergence …
A) In probability
B) with probability 1
C) in distribution
D) in rth mean

21) Consider the following statements:
(I) For a positive recurrent state the mean recurrence time is finite
(II) For a null recurrent state the mean recurrence time is infinite.

Which of the following are correct?

A) Only I is correct
B) Only II is correct
C) Both I and II are correct
D) Neither I nor II is correct

22) Given P = $\begin{bmatrix} .25 & .50 & .25 \\ 0 & 0 & 1 \\ .65 & .35 & 0 \end{bmatrix}$ and the initial probabilities as $P(X_0 = i) = \frac{1}{3}, i = 1,2,3$. What is the value of $P(X_1 = 1)$?

A) 0.1
B) 0.2
C) 0.3
D) 0.4

23) For a Birth and death process with $\lambda_n = \lambda$ and $\rho_n = n\rho$, the $P[X(t) = n]$ is
A) Poisson (1-ρ)
B) Poisson (nρ)
C) Poisson (ρ)
D) Poisson (n-ρ)

24) Suppose we have a Poisson stream of customers entering a service facility with two servers. Now consider the following statements:

(I) a waiting system in which lines form in front of each teller.
(II) A single line served by the two tellers.

Which of the following are better?

A) Only (I) is better
B) Only (II) is better
C) Both (I) and (II) are good
D) Neither (I) nor (II) is good

25) If F(t) = t/n, then the renewal function M(t) satisfies the relation

A) $M(t) = n\int_0^t F(t-x)dF(x)$
B) $nM(t) = t + \int_0^t F(t-x)dF(x)$
C) $nM(t) = t + \int_0^t M(t-x)dx$
D) $M(t) = n + \int_0^t F(t-x)dM(x)$

26) The sufficient statistics based on n observations correspond to the parameters in Beta (a, b) is

A) $(\sum x_i, \sum {x_i}^2)$
B) $\left(\sum x_i, \sum(1-x_i)\ \right)$
C) $(\sum x_i, \prod x_i)$
D) $(\sum x_i, \prod(1-x_i))$

27) Given the following statements about a one parameter exponential family of distribution:
It always admits sufficient statistics.

(I) The moment estimator $\hat{\theta}$ based on sufficient statistics is CAN for θ.
(II) The asymptotic variance attains CRLB.

Which of the following are correct?

A) Only I and II are correct
B) Only I and III are correct
C) Only II and III are correct
D) All are correct

28) If 3, 8, 5, 4 and 10 are exponential samples with mean θ. The Fisher information function evaluated at θ=2 is

A) 0.50
B) 0.80
C) 1.20
D) 1.25

29) Consider the following statements:

(I) Least square estimators are unbiased in case of nonlinear models
(II) Least square estimators doesn't satisfy any large sample properties

Which of the following are correct?

A) Only I is correct.
B) Only II is correct
C) Both I and II is correct
D) Neither I nor II is correct

30) If f(x) is any family of parametric distribution and g(x) be any function independent of the parameter, then g(x) to be complete

A) the parametric function is complete
B) the function g(x) is a one-to-one function of the parametric function
C) the function g(x) is continuous
D) the function g(x) is linear

31) A sample of size 1 is taken from $b(1,\theta)$. Suppose the test function used to test $H_0: \theta = 1/2$ against $H_1: \theta = 3/4$ is

$$\phi(x) = \begin{cases} 1 & if\ x > 1 \\ 0.6, & if\ x = 1 \\ 0, & if\ x < 1 \end{cases}$$

Then the power of the test will be

A) 0.3
B) 0.55
C) 0.45
D) 0.4

32) For testing $H: \theta \le \theta_1\ or\ \theta \ge \theta_2\ (\theta_1 < \theta_2)$ against $K: \theta_1 < \theta < \theta_2$, the power function

A) is strictly increasing from $\theta_1$to θ_2
B) is strictly decreasing from $\theta_1$to θ_2
C) attains the maximum at $\theta_0 \in (\theta_1, \theta_2)$
D) attains the maximum at $\theta_1 \in (\theta_1, \theta_2)$

33) A UMP unbiased test for a parameter θ of interest in (K+1) parameter exponential family, treating the remaining parameters as nuisance parameters, where U is a sufficient statistic for θ and $\underline{T}$ is a sufficient statistic for the nuisance parameters, can be derived provided

A) the distribution of $U \mid \underline{T} = \underline{t}$ is continuous
B) the distribution of $\underline{T} \mid U = u$ is continuous
C) the distribution of $U \mid \underline{T} = \underline{t}$ is one-parameter exponential
D) the distribution of $\underline{T} \mid U = u$ is one-parameter exponential.

34) Let θ be the parameter of the distribution which belongs to one-parameter exponential family. Then for testing $H: \theta \leq \theta_1 \; or \; \theta \geq \theta_2 (\theta_1 < \theta_2)$ ag. $K: \theta_1 < \theta < \theta_2$, there exists
A) MP test
B) UMP test
C) UMP unbiased test
D) Neyman structure test

35) Let $x_1, x_2, \ldots, x_m$ be a random sample from $N(\theta, 1)$. Then for testing $H_0: \theta = \theta_0$ against $H_1: \theta \neq \theta_0$,
A) there exists a UMP test which is the same as the likelihood ratio test
B) there exists a UMP unbiased test which is the same as the likelihood ratio test
C) there exists a UMP test which is the different from the likelihood ratio test
D) there exists a UMP unbiased test which is different from the likelihood ratio test

36) Which of the following is not true for Wilcoxon's Rank Sum test?
A) This test applies to the paired replicates samples
B) In this test, the parameter of interest is the unknown shift in location
C) The test statistic is the sum of the ranks assigned to the treatment group
D) This test can be used even when the actual magnitudes of the observations are not available, but only the ranks are available.

37) The large sample standard deviation of Wilcoxon-Mann-Whitney Rank-sum test correspond to m = 5 and n = 8 is
A) 3.56
B) 4.21
C) 4.47
D) 5.32

38) Consider the following statements:
(I) In Mann-Whitney test, one tests the identicalness of two random variables
(II) In Sign test for paired samples we often find the difference between the paired values

Which of the following are correct?

A) Only I is correct.
B) Only II is correct
C) Both I and II is correct
D) Neither I nor II is correct

39) Given

X	145	146	157	156
Y	147	148	144	158

If samples X and Y are in pairs and are independent, and are drawn from a continuous bivariate population, then what is the value of the Kendall's statistic, K.
A) 0
B) 1
C) 2
D) 3

40) A Kruskal-Wallis's test involving three groups with 7, 5, and 6 observations has ranks 75.5, 48.5, and 47 respectively. What is the value of Kruskal-Wallis's test?
A) 0.345
B) 0.444
C) 0.678
D) 0.998

41) Consider the $N_p(\underline{\mu}, \Sigma)$ population. Let S be the MLE of Σ based on a random sample of size n. Then T^2 statistic for testing H: $\underline{\mu} = \underline{0}$ is given by
A) $\underline{\bar{X}}' S^{-1} \underline{\bar{X}}$
B) $n(n-1)\, \underline{\bar{X}}'^{S^{-1}\underline{\bar{X}}}$
C) $n\, \underline{\bar{X}}' S^{-1} \underline{\bar{X}}$
D) $(n-1)\, \underline{\bar{X}}' S^{-1} \underline{\bar{X}}$

42) Suppose $\underline{X} \sim N_p(0, \boldsymbol{I})$ and $S \sim W_p(I, n)$ and are independent. Then Which of the following transforms follows a Hotelling's T^2 distribution?
A) $\underline{X}' S^{-1} \underline{X}$
B) $n(n-1)\, \underline{X}' S^{-1} \underline{X}$
C) $n\, \underline{X}' S^{-1} \underline{X}$
D) $\frac{n-p}{(n-1)p}\, \underline{X}' S^{-1} \underline{X}$

43) Behrens-Fisher problem is
A) the problem of comparing means of two independent normal populations with unequal known variances
B) the problem of comparing means of two independent normal populations with unequal unknown variances
C) the problem of comparing means of two d'ependent normal populations with unequal known variances
D) the problem of comparing means of two dependent normal populations with unequal unknown variances

44) Suppose $S_1 \sim W_p(\Sigma_1, n_1)$, and $S_2 \sim W_p(\Sigma_2, n_2)$ are independent Wishart matrices. Then $S_1 + S_2$ is
A) Always a Wishart matrix
B) a Wishart matrix when $\Sigma_1 = \Sigma_2$
C) a Wishart matrix when $n_1 = n_2$
D) a Wishart matrix when $n_1 = n_2$ and $\Sigma_1 = \Sigma_2$

45) Suppose for p×1 vector $\underline{X}$, $V(\underline{X}) = \Sigma$. Then $V\left(\sum_{i=1}^{p} X_i\right)$ is given by

(A) $\underline{\mathbf{1}}^T \Sigma \, \underline{\mathbf{1}}$
(B) trace (Σ)
(C) $\underline{\mathbf{1}} \, \Sigma \, \underline{\mathbf{1}}^T$
(D) $|\Sigma|$

46) Suppose a random variable X has pdf

$$f(x) = \begin{cases} \frac{1}{\Gamma(\alpha)\beta^{\alpha}} e^{-x/\beta} x^{\alpha-1}, & x > 0 \\ 0 , & otherwise \end{cases}$$

Then which of the following statements is not true?
A) $E(X)$ exists for every $\alpha > 0$ and $\beta > 0$
B) $V(X)$ exists for every $\alpha > 0$ and $\beta > 0$
C) $E(1/X)$ exists for every $\alpha > 0$ and $\beta > 0$
D) All above statements are true.

47) Suppose $(X, Y)'$ follows a bivariate normal distribution with following parameters:
$\mu' = (2.5, 12.2)$ and $\Sigma = \begin{pmatrix} 5 & 2 \\ 2 & 3 \end{pmatrix}$
then the equation of regression line of X on Y is
A) x - 0.4 y +2.38 = 0
B) x - 0.667 y +5.6374 = 0
C) 0.4 x – y + 11.2 = 0
D) 0.667 x – y + 10.5325 = 0

48) Consider the simple linear regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i, \quad i = 1,2,\ldots,n$, where β_0 and β_1 are unknown parameters, ϵ_i's are uncorrelated random errors with mean 0 and finite variance $\sigma^2 > 0$. Let $\hat{\beta}_i$ be the least squares estimator of $\beta_i, i = 0, 1$. Consider the following statements:

(I) A 95% joint confidence region for (β_0, β_1) is the region bounded by an ellipse.
(II) The expression for covariance between $\hat{\beta}_0$ and $\hat{\beta}_1$ does not involve σ^2.

Which of the above statements is/are true?
A) Only (I)
B) Only (II)
C) Both (I) and (II)
D) Neither (I) nor (II)

49) For a linear regression model fitted using least squares approach , let $\hat{y}_i$ denote the fitted values and e_i dente the residuals. Then which of following statements is always true?
(I) $\sum_{i=0}^{n} \hat{y}_i e_i = 0$
(II) $\sum_{i=0}^{n} e_i = 0$

A) Only (I)
B) Only (II)
C) Both (I) and (II)
D) Neither (I) nor (II)

50) Which of the following is an example of linear regression model?
(I) $y_i = \alpha_1 + \gamma_1 x_1 + (\alpha_1 - \alpha_1) x_2 + (\gamma_1 - \gamma_1) x_1 x_2 + \epsilon_i$
(II) $y_i = \alpha_1 + \gamma_1 x_1 + \gamma_2 x_2 + \gamma_1 \gamma_2 x_3 + \epsilon_i$

A) Only (I)
B) Only (II)
C) Both (I) and (II)
D) Neither (I) nor (II)

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Rough Work: