THE MAHARAJA SAYAJIRAO UNIVERSITY OF BARODA, VADODARA

Ph. D. ENTRANCE TEST (PET) 2025

Signature of Invigilator

Roll. No. | | | | | | |

Paper - II
Mathematical Sciences

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Maximum Marks: 50 **No. Of Printed Pages: 8**

Instruction for the Candidate:

1. This paper consists of **FIFTY (50)** multiple choice type questions. Each Question carries **ONE (1)** mark.
2. There is no Negative Marking for Wrong Answer.
3. A separate OMR Answer Sheet has been provided to answer questions. Your answers will be evaluated based on your response in the OMR Sheet only. No credit will be given for any answering made in question booklet.
4. Defective question booklet or OMR if noticed may immediately replace by the concerned invigilator.
5. Write roll number, subject code, booklet type, category and other information correctly in the OMR Sheet else your OMR Sheet will not be evaluated by machine.
6. Select most appropriate answer to the question and darken appropriate oval on the OMR answer sheet, with black / blue ball pen only. DO NOT USE PENCIL for darkening. In case of over writing on any answer, the same will be treated as invalid. Each question has exactly one correct answer and has four alternative responses (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example:Ⓐ ● Ⓒ Ⓓ where (B) is correct response.
7. Rough Work is to be done in the end of this booklet.
8. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
9. Calculators, Log tables any other calculating devices, mobiles, slide rule, text manuals etc are NOT allowed in the examination hall. If any of above is seized from the candidates during examination time; he/ she will be immediately debarred from the examination and corresponding disciplinary action will be initiated by the Center Supervisor as deemed fit.
10. DO NOT FOLD or TEAR OMR Answer sheet as machine will not be able to recognize torn or folded OMR Answer sheet.
11. **You have to return the OMR Answer Sheet to the invigilator at the end of the examination compulsorily** and must not carry it with you outside the Examination Hall. You are however, allowed to carry original question booklet on conclusion of examination.

Paper - II
Mathematical Sciences

Note: This paper contains **FIFTY (50)** multiple-choice questions. Each Question carries **ONE (1)** ma

01) There is no vector space of cardinality
A) 151
B) 121
C) 141
D) 289

02) Number of cyclic groups that have only odd number of generators is
A) 2
B) 1
C) 0
D) ∞

03) Let $\mathbf{f}: [0,1] \to [0,1]$ be a function such that $|\mathbf{f}(x) - \mathbf{f}(y)| \leq |x - y|$, for all $x, y \in [0,1]$
Then
A) $\mathbf{f}$ has only one fixed point.
B) $\mathbf{f}$ has uncountable fixed points.
C) $\mathbf{f}$ has either one or uncountable fixed points.
D) there exists $\mathbf{f}$ such that f has no fixed point.

04) Which of the following is a field?
A) $\mathbb{Z}$ (the integers)
B) $\mathbb{Z}_6$ (integers mod 6)
C) $M_2(\mathbb{R})$ (2×2 real matrices)
D) $\mathbb{Q}$ (rational numbers)

05) Let A be nxn real idempotent matrix. Then which of the following is true?
A) All eigenvalues of A are 1
B) All eigenvalues of A are either 0 or 1
C) A is invertible
D) All eigenvalues of A are either i or $-i$

06) With respect to the usual metric over $X = [0,1) \cup \{2\}$, which of the following is open as well as closed set in X?
A) (0.5, 1)
B) $\{2\}$
C) $\{0\}$
D) (0, 0.5)

07) Let $\{x_n\}$ and $\{y_n\}$ be any two Cauchy but not convergent sequences in a metric space (X, d), then which of the following statements is true?
A) $\{d(x_n, y_n)\}$ is Cauchy but not convergent.
B) $\{d(x_n, y_n)\}$ need not be a Cauchy sequence.
C) $\{d(x_n, y_n)\}$ need not be a bounded sequence.
D) $\{d(x_n, y_n)\}$ is always a convergent sequence.

08) Let f be a real continuous map on X=[0,100]. Then
A) f is uniformly continuous, f(X) is compact and connected.
B) f(X) is compact and connected, f is continuous but need not be uniformly continuous.
C) f is uniformly continuous, f(X) is compact but need not be connected.
D) f is uniformly continuous, f(X) is connected but need not be compact.

09) $\lim_{n\to\infty} \frac{1}{n^{m-1}}((n+1)^m + (n+2)^m + \ldots + (n+k)^m - kn^m) =$
A) $\frac{mk(k+1)}{2}$
B) $\frac{mk}{2}$
C) 0
D) ∞

10) Let G be a group of order n. Which of the following implies that G is abelian?
A) n=15
B) n=21
C) n= 36
D) n=81

11) Let f be a complex-valued function in the open unit disc D of the complex plane C, if
A) f^2 and f^6 analytic, then f is analytic
B) f^2 and f^4 analytic, then f is analytic
C) f^2 and f^3 analytic, then f is analytic
D) f^3 and f^6 analytic, then f is analytic

12) 12. Let the function f be analytic in the entire complex plane, and suppose that $\frac{f(Z)}{Z} \to 0$ as $|z| \to \infty$, then
A) f is constant
B) f is polynomial of degree 1
C) f is polynomial of degree ≥ 1
D) None of the above

13) The set of all points of convergence of the sequence $\left\{\frac{1}{n}\right\}$ with respect to the co-finite topology over R, the real line,
A) is {0}
B) is R, all real numbers
C) is $\emptyset$
D) is {0 ,1}

14) The sequence of functions $f_n(x) = x^{1/n}, \forall x \in [0,1], (n = 1,2,3,\dots)$, is
   A) not pointwise convergence at every point of [0,1]
   B) convergent at only finitely many points of [0,1]
   C) is uniformly convergent over [0,1]
   D) pointwise convergent but not uniformly convergent over [0,1]

15) Function $f(x) = sin(3x) + cos(6x), \forall\, x \in \mathbb{R}$, is
   A) not a periodic function
   B) a periodic function with period π
   C) a periodic function with period $\frac{2\pi}{3}$
   D) a periodic function with period $\frac{\pi}{3}$

16) Let $\{a_n\}$ (n=1, 2, 3, ...) be a sequence of non-negative defined as $a_{n+1} = (a_n - 1)^3 + 1$, (n=1,2,3,...). Then $\{a_n\}$
   A) converges to 1, if $0 \leq a_1 \leq 2$
   B) monotonically decreasing and converges to 1, if $1 < a_1 < 2$
   C) monotonically increasing and converges to 1, if $1 < a_1 < 2$
   D) divergent, if $1 < a_1 < 2$

17) Given that the sum of two of its roots is zero. Then the solutions of the equation $6x^4 - 3x^3 + 8x^2 - x + 2 = 0$ are
   A) $x = \pm\frac{i}{\sqrt{3}},\ x = \frac{1}{4} \pm \frac{\sqrt{15}}{4}\ i$
   B) $x = \pm\frac{i}{\sqrt{5}},\ x = \frac{1}{4} \pm \frac{\sqrt{13}}{4}\ i$
   C) $x = \pm\frac{i}{\sqrt{5}},\ x = \frac{1}{2} \pm \frac{\sqrt{15}}{2}\ i$
   D) $x = \pm\frac{i}{\sqrt{3}},\ x = \frac{1}{2} \pm \frac{\sqrt{15}}{2}\ i$

18) Which of the following statements is TRUE?
   A) Every Cauchy sequence in a metric space is convergent.
   B) $\{a_n\}$ is a Cauchy sequence in $\mathbb{R}$ $\Leftrightarrow$ It has at least one convergent subsequence.
   C) If $|\, a_{n+1} - a_n| \to 0$ as $n \to \infty$ in $\mathbb{R} \Rightarrow \{a_n\}$ is a Cauchy sequence in $\mathbb{R}$.
   D) $\{a_n\}$ is a Cauchy sequence in $\mathbb{R} \Rightarrow |\, a_{n+1} - a_n| \to 0$ as $n \to \infty$.

19) The Cantor set is
   A) closed but not compact set in $\mathbb{R}$.
   B) a perfect and compact set in $\mathbb{R}$.
   C) closed but not perfect set.
   D) neither compact nor perfect set in $\mathbb{R}$.

20) Let $\{a_n\}$ (n=1, 2, 3, ...) be sequence of positive real numbers. Which of the following is TRUE?
   A) Convergence of $\sum \frac{\sqrt{a_n}}{n}$ implies the convergence of $\sum a_n$
   B) Convergence of $\sum \frac{a_n}{\sqrt{n}}$ implies the convergence of $\sum a_n$
   C) Convergence of $\sum a_n$ implies the convergence of $\sum \frac{\sqrt{a_n}}{n}$
   D) Convergence of $\sum a_n$ implies the convergence of $\sum \sqrt{\frac{a_n}{n}}$

21) The series $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$ converges to
   A) $\frac{\pi}{4}$
   B) $\frac{\pi}{2}$
   C) π
   D) $2\,\pi$

22) Let (X, d) be a complete metric space and T:X→X satisfies the condition
   d(T(x),T(y)) < d(x, y), ∀x, y∈X, x≠y. Then T
   A) is a contraction on X.
   B) has at least one fixed point.
   C) has exactly one fixed point.
   D) has at the most one fixed point.

23) Total variation of the function f(x)=Cos($\frac{x}{2}$) over the interval [0,π] is
   A) 1
   B) 2
   C) 3
   D) 4

24) In the inner product space C[0, 1], define $\langle f, g\rangle = \int_0^1 f(t)g(t)\,dt$. Which of the following functions is orthogonal to f(t)=t?
   A) g(t)=1
   B) g(t)=t^2
   C) g(t)=sin(πt)
   D) g(t)=t−$\frac{2}{3}$

25) Which of the following statements is FALSE?
   A) Product of two orthogonal matrices is orthogonal.
   B) Inverse of a non-singular Hermitian matrix is Hermitian.
   C) Product of two unitary matrices is unitary matric.
   D) Product of two Hermitian metrices is Hermitian metric.

26) Let $f(z) = |z|^2$, then which of the following is true?

A) f is analytic everywhere
B) f satisfies C-R equations at origin but is not analytic
C) f is differentiable only on real axis
D) f does not satisfy C-R equations anywhere.

27) What is the value of $\int_{|z|=2} \frac{z^2+1}{z-1} dz$?

A) 0
B) $2\pi\, i$
C) $4\pi i$
D) $2\pi i(1+i)$

28) Which of the following is the value of the integral $\int_C \frac{1}{(z-1)(z-2)} dz$, where C represents a circle with center O and radius 3?

A) 0
B) $2\pi\, i$
C) $4\pi\, i$
D) $6\pi\, i$

29) The Taylor series expansion of $f(z) = \frac{1}{1-z}$ about $z = 0$ and the region of convergence are

A) $\sum_{n=0}^{\infty} z^n \text{ and } |z| < 1$
B) $\sum_{n=1}^{\infty} z^n \text{ and } |z| < 1$
C) $\sum_{n=0}^{\infty} z^n \text{ and } |z| > 1$
D) $\sum_{n=1}^{\infty} \frac{z^n}{n} \text{ and } |z| < 1$

30) The map $f(z) = z^2$ is conformal at all points except

A) $z = 0$
B) $z = 1$
C) $z = i$
D) $z = -i$

31) Let $x_0 = 1.5$. If we apply one iteration of Newton-Raphson method to $f(x) = x^2 - 2$, then what is the first approximation x_1?

A) 1.4167
B) 1.5
C) 1.25
D) 1.35

32) Given the points (1, 1), (2, 4) and (3, 9). Then the Lagrange's interpolating polynomial $P(x)$ is

A) x^2
B) $x^2 + 1$
C) $2x^2$
D) $x^2 - x + 1$

33) Simpson's 1/3rd rule is applicable only when the number of subintervals n is

A) Odd
B) Even
C) Prime
D) Multiple of 3

34) Which of the following is an approximate solution of $\frac{dy}{dx} = y + x$, $y(0) = 1$ at $x = 0.1$ using Euler's method with step size $h = 0.1$?

A) 1.1
B) 1.11
C) 1.12
D) 1.105

35) The Fredholm integral equation of the second kind is of the form $y(x) = f(x) + \lambda \int_a^b K(x,t) y(t) dt$ This equation becomes Volterra equation if

A) $K(x,t) = 0$
B) The upper limit of integral is x
C) $f(x) = 0$
D) $\lambda = 0$

36) Which of the following is true for Kernal $K(x,t) = sin(x+t)$ of an integral equation?

A) Separable
B) Anti-symmetric
C) Constant
D) Discontinuous

37) For Volterra integral equations, the resolvent kernel $R(x,t)$ satisfies

A) $R(x,t) = K(x,t) + \int_t^x K(x,s) R(s,t) ds$
B) $R(x,t) = \delta(x-t)$
C) $R(x,t) = \int_0^x K(s,t) R(x,s) ds$
D) $R(x,t) = 0$

38) The IVP $y' = 4y^{3/4}, y(0) = 0$ has

A) uncountable solutions
B) no solution
C) two solutions
D) a unique solution

39) Let y_1 and y_2 be solutions of the differential equation $y'' + p(x)y' + q(x)y = 0$ on an open interval *I* with Wronskian $W(y_1, y_2)(x)$ and p and q are continuous on *I*. If y_1 and y_2 have maxima or minima at the same point on *I*, then

A) $W(y_1, y_2)(x) \equiv 0$ on *I* and $\{y_1, y_2\}$ forms a fundamental set of solutions
B) $W(y_1, y_2)(x) \neq 0$ on *I* and $\{y_1, y_2\}$ form a fundamental set of solutions
C) $W(y_1, y_2)(x) \equiv 0$ on *I* and $\{y_1, y_2\}$ does not form a fundamental set of solutions
D) $W(y_1, y_2)(x) \neq 0$ on *I* and $\{y_1, y_2\}$ does not form a fundamental set of solutions

40) If $y(x) = c_1 x + c_2 e^{-x}$ is the general solution of the homogeneous part of the equation $(x+1)y'' + xy' - y = (x+1)^2$, then which of the following is not a particular integral?
A) $x^2 + 1$
B) $(x+1)^2$
C) $x^2 - 1$
D) $(x-1)^2$

41) The eigen value $\lambda > 0$ for the boundary value problem $y'' + \lambda y = 0, y(0) = 0,\ y(1) + y'(1) = 0$ satisfy the equation
A) $\sqrt{\lambda} + tan\sqrt{\lambda} = 0$
B) $\sqrt{\lambda} + tanh\sqrt{\lambda} = 0$
C) $\sqrt{\lambda} + sin\sqrt{\lambda} = 0$
D) $\sqrt{\lambda} + cos\sqrt{\lambda} = 0$

42) The solution of the PDE $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = z$ with $z(1, y) = y^2$ is
A) $z(x,y) = \frac{x}{x^2+y^2}$
B) $z(x,y) = \frac{x^2}{y}$
C) $z(x,y) = \frac{x^2+y^2}{x}$
D) $z(x,y) = \frac{y^2}{x}$

43) The second order PDE
$\frac{\partial^2 z}{\partial x^2} - 2(sinx)\frac{\partial^2 z}{\partial x \partial y} - (cos^2 x)\frac{\partial^2 z}{\partial y^2} - (cosx)\frac{\partial z}{\partial y} = 0$ is
A) hyperbolic only in the first quadrant $x > 0, y > 0$
B) hyperbolic in the complete plane
C) parabolic in the complete plane
D) elliptic in the complete plane

44) The general solution of the PDE
$\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} + 2\frac{\partial z}{\partial x} + 2\frac{\partial z}{\partial y} + z = 0$ is

A) $z = e^{-x}[\phi(y-x) + x\psi(y+x)]$
B) $z = e^{-x}[\phi(y-x) + x\psi(y-x)]$
C) $z = e^{x}[\phi(y-x) + x\psi(y+x)]$
D) $z = e^{x}[\phi(y-x) + x\psi(y-x)]$

45) For the PDE $x^2 u_{xx} - y^2 u_{yy} = x^2 y^2 + x$, $x > 0$, the equations of the characteristic curves are
A) $xy = c_1,\ x + y = c_2$
B) $xy = c_1,\ \frac{y}{x} = c_2$
C) $x^2 + y^2 = c_1,\ x^2 - y^2 = c_2$
D) None of these

46) Which of the following transformations does not affect the equation of motion derived from a Lagrangian?
A) $L \to L + \alpha\ddot{q}$
B) $L \to L + \beta\dot{q}^2$
C) $L \to L + \gamma q\dot{q}$
D) $L \to L + \frac{d}{dt}F(q,t)$

47) If the Lagrangian of a particle is $L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$, what is the equation of motion from Hamilton's principle?
A) $m\dot{x} + kx = 0$
B) $m\ddot{x} + kx = 0$
C) $\dot{x}^2 = kx^2$
D) $\ddot{x} = kx$

48) The Lagrangian of a particle of mass m moving in one direction is $L = \frac{1}{2}m(\dot{x}^2 - \omega^2 x^2)e^{\gamma t}$, where all constants are positive. Then the Hamiltonian of the particle is
A) $H = \frac{p^2}{2m}e^{-\gamma t} + \frac{1}{2}m\omega^2 x^2 e^{\gamma t}$
B) $H = \frac{p^2}{2m}e^{-\gamma t} - \frac{1}{2}m\omega^2 x^2 e^{\gamma t}$
C) $H = \frac{p^2}{2m}e^{\gamma t} + \frac{1}{2}m\omega^2 x^2 e^{-\gamma t}$
D) $H = \frac{p^2}{2m}e^{\gamma t} - \frac{1}{2}m\omega^2 x^2 e^{-\gamma t}$

49) The extremal of the functional $J[y(x)] = \int_0^2 \left(\frac{y'^2}{x}\right) dx$,$y(0) = \alpha,\ y(2) = \beta$, is a parabola passing through the origin, then α and β are
A) $\alpha = 0,\ \beta = 1$
B) $\alpha = 1,\ \beta = 2$
C) $\alpha = -1,\ \beta = 2$
D) None of these

50) The variational problem of extremizing the functional $J[y(x)] = \int_0^{2\pi}[(y')^2 - y^2]dx$, $y(0) = 1, y'(2\pi) = 1$ has
A) has a unique solution
B) exactly two solutions
C) no solution
D) infinite number of solutions

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Rough Work: