THE MAHARAJA SAYAJIRAO UNIVERSITY OF BARODA, VADODARA

Ph. D. ENTRANCE TEST (PET) – 7th August 2022

Signature of Invigilator

Roll. No. | | | | | | |

Paper - II
Statistics (22/46)

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Maximum Marks: 50 **No. Of Printed Pages: 8**

Instruction for the Candidate:

1. This paper consists of **FIFTY (50)** multiple choice type questions. Each Question carries **ONE (1)** mark.
2. There is no Negative Marking for Wrong Answer.
3. A separate OMR Answer Sheet has been provided to answer questions. Your answers will be evaluated based on your response in the OMR Sheet only. No credit will be given for any answering made in question booklet.
4. Defective question booklet or OMR if noticed may immediately replace by the concerned invigilator.
5. Write roll number, subject code, booklet type, category and other information correctly in the OMR Sheet else your OMR Sheet will not be evaluated by machine.
6. Select most appropriate answer to the question and darken appropriate oval on the OMR answer sheet, with black / blue ball pen only. DO NOT USE PENCIL for darkening. In case of over writing on any answer, the same will be treated as invalid. Each question has exactly one correct answer and has four alternative responses (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.
   Example:Ⓐ ⬤ Ⓒ Ⓓ where (B) is correct response.
7. Rough Work is to be done in the end of this booklet.
8. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
9. Calculators, Log tables any other calculating devices, mobiles, slide rule, text manuals etc are NOT allowed in the examination hall. If any of above is seized from the candidates during examination time; he/ she will be immediately debarred from the examination and corresponding disciplinary action will be initiated by the Center Supervisor as deemed fit.
10. DO NOT FOLD or TEAR OMR Answer sheet as machine will not be able to recognize torn or folded OMR Answer sheet.
11. **You have to return the OMR Answer Sheet to the invigilator at the end of the examination compulsorily** and must not carry it with you outside the Examination Hall. You are however, allowed to carry original question booklet on conclusion of examination.

Paper - II
Statistics (22/46)

Note: This paper contains **FIFTY (50)** multiple-choice questions. Each Question carries **ONE (1)** mark.

01) Let $f_n(x) = x^n, 0$

10) A sequence of jointly distributed random variables $\{X_n\}_{n=1}^{\infty}$ having finite means is said toobey the strong law of large numbers if

A) $\frac{1}{n} \quad {}_{k=1}^{n} X_k \quad \frac{1}{n} \quad {}_{k=1}^{n} E(X_k) \quad 0$ in probability as n

B) $\frac{1}{n} \quad {}_{k=1}^{n} X_k \quad {}_{k=1}^{n} E(X_k) \quad 0$ in mean square as$n$

C) $\frac{1}{n} \quad {}_{k=1}^{n} X_k \quad \frac{1}{n} \quad {}_{k=1}^{n} E(X_k) \quad 0$ with probability 1 as n

D) $\frac{1}{n} \quad {}_{k=1}^{n} X_k \quad {}_{k=1}^{n} E(X_k) \quad 0$ with probability 1 as n

11) A news magazine publishes 3 columns entitled “Art”, “Books” and “Cinema”. Let A be the event that a reader reads “Art” column regularly, B be the event that a reader reads “Books” column regularly and C be the event that a reader reads “Cinema” column regularly. Reading habits of a randomly selected reader with respect to these 3 columns are assumed to have the following probabilities:

Event:	A	B	C				
Probability:	0.14	0.23	0.37	0.08	0.09	0.13	0.05

Then the probability that the randomly selected individual regularly reads the Art column given that he or she regularly reads at least one of the other two columns is

A) 0.348
B) 0.286
C) 0.384
D) 0.255

12) Let $\{X_n\}_{n=1}^{\infty}$ be a sequence of random variables where $X_n \sim Geom\left(\frac{\lambda}{n}\right)$, $\lambda > 0$. Then the sequence of random variables $\{Y_n\}_{n=1}^{\infty}$, where $Y_n = \frac{X_n}{n}$ converges

A) in distribution to $Exp(n\,\lambda)$
B) in probability to $Exp(n\,\lambda)$
C) in distribution to $Exp(\lambda)$
D) in probability to $Exp(\lambda)$

13) If $X_k \sim b(1\,, p), for\ k = 1, 2, \ldots, n;$ then using Chebyshev’s form of weak law of large numbers, which of the following statements is true?

A) $\lim_{n \quad \infty} P\,[| \quad {}_{k=1}^{n} X_k \quad p\,| < \varepsilon] = 1$

B) $\lim_{n \quad \infty} P\left[\left|\frac{1}{n} \quad {}_{k=1}^{n} X_k \quad p\right| < \varepsilon\right] = 1$

C) $\lim_{n \quad \infty} P\,[| \quad {}_{k=1}^{n} X_k \quad p\,| > \varepsilon] = 0$

D)) $\lim_{n \quad \infty} P\,[| \quad {}_{k=1}^{n} X_k \quad n\,p\,| < \varepsilon] = 1$

14) The following UMP unbiased test was obtained based on a random sample of size 25 from N(θ , 16) for testing $H{:}\,\theta = 10$ ag. $K{:}\,\theta \quad 10$:

$\phi(x) = 1\ if\ \bar{X} < 8.432\ or\ \bar{X} > 11.568$
$= 0\ ot\ erwise$

The power of the test for $\theta = 6$ will be approximately

A) 0.998
B) 0.002
C) 0.95
D) 0.852

15) For testing $H{:}\,\theta = \theta_0$ ag. $K{:}\,\theta = \theta_1$using SPRT, the correct expression for ASN function, in usual notations, is

A) $\frac{\beta(\theta) \ln B - (1 - \beta(\theta)) \ln A}{E(Z)}$

B) $\frac{\beta(\theta) \ln B + (1 - \beta(\theta)) \ln A}{E(Z)}$

C) $\frac{\beta(\theta) \ln A - (1 - \beta(\theta)) \ln B}{E(Z)}$

D) $\frac{\beta(\theta) \ln A + (1 - \beta(\theta)) \ln B}{E(Z)}$

16) For testing a hypothesis $H_0{:}\,\pi = \pi_0$ where π denotes the probability of success for a Binomial probability law, the Score test statistic will be of the form

A) $\frac{\hat{\pi} - \pi_0}{\sqrt{\hat{\pi}\,(1-\hat{\pi})}}$

B) $\frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\pi_0\,(1-\pi_0)}{n}}}$

C) $\frac{\hat{\pi} - \pi_0}{\sqrt{\frac{\hat{\pi}\,(1-\hat{\pi})}{n}}}$

D) $\frac{\pi - \pi_0}{\sqrt{\frac{\hat{\pi}\,(1-\hat{\pi})}{n}}}$

17) In context with the Kolmogorov-Smirnov goodness of fit test for single sample for the two-sided alternative,

A) $D_n^+ = \underset{n}{Sup}[F(x) \quad F_n(x)]$, $D_n^- = \underset{n}{Sup}[F_n(x) \quad F(x)]$

B) $D_n^+ = \underset{n}{Sup}[F_n(x) \quad F(x)]$, $D_n^- = \underset{n}{Inf}[F(x) \quad F_n(x)]$

C) $D_n^+ = \underset{n}{Sup}[F_n(x) \quad F(x)]$, $D_n^- = \underset{n}{Sup}[F(x) \quad F_n(x)]$

D) $D_n^+ = \underset{n}{Inf}[F(x) \quad F_n(x)]$, $D_n^- = \underset{n}{Sup}[F_n(x) \quad F(x)]$

18) In a certain region, 100 animals from an animal population that were thought to be near extinction were caught, tagged and released to mix into the animal population. After they had an opportunity to mix, a random sample of 40 animals from the animal population was recaptured. Let X: the number of tagged animals in the recaptured sample. If in a recaptured sample, 16 tagged animals out of 100 were

captured, then the estimated animal population in the region would be

A) 640

B) 1600

C) 250

D) 800

19) Let $X_1, X_2, \dots, X_n$ be a random sample drawn from $N(\theta, \sigma^2)$ where θ is unknown but σ^2 is known. Then the UMVUE of $g(\theta) = \theta^2$ is

A) $\bar{X}^2$

B) $\bar{X}^2 \quad \frac{\sigma}{n}$

C) $\frac{\bar{X}^2}{\sigma^2}$

D) $\bar{X}^2 \quad \frac{\sigma^2}{n}$

20) Three children, denoted by A, B and C, arranged in a circle play a game of throwing a ball to one another. At each stage the child having the ball is equally likely to throw it to any one of the other two children. Suppose that X_0 denotes the child who had the ball initially and $X_n, n \quad 1$ denotes the child who had the ball after n throws. Then for a Markov chain $\{X_n, n \quad 1\}$,

A) $P(X_2 = B \mid X_0 = C) =$

B) 1/2

C) 3/8

D) 1/4

E) 3/4

21) Suppose $X \sim \exp(1)$ and let $Y = X^{1 \ \alpha}$. Then $E(Y) =$

A) $(1 + 1 \ \alpha)$

B) $(1 \ \alpha)$

C) $(1 \ \alpha)^2$

D) $\{ \ (1 + 1 \ \alpha)\}^2$

22) Let denote the order statistics for a random sample of size 5 from $U(0, 1)$. Let = Then the probability density function of will be

A) $g_R(r) = n(n \quad 1)r^{n-1}; 0 < r < 1$

B) $g_R(r) = n(n \quad 1)r^{n-1}(1 \quad r); 0 < r < 1$

C) $g_R(r) = n \ r^{n-2}(1 \quad r); 0 < r < 1$

D) $g_R(r) = n(n \quad 1)r^{n-2}(1 \quad r); 0 < r < 1$

23) For some fitted logistic regression model, $\hat{\beta}_0 =$ 5.309, $\widehat{SE}(\hat{\beta}_0) = 1.1337$, $\hat{\beta}_1 = 0.111$, $\widehat{SE}(\hat{\beta}_1) =$ 0.0241. Then the $100(1 \quad \alpha)\%$ confidence interval for β_0 when the critical value is 1.96, will be

A) (-7.531, -3.087)

B) (0.064, 0.158)

C) (-7.828, -2.789)

D) (0.109, 0.112)

24) Let $= (0, \quad)$, let $\mathcal{a}$ be the real line, and let $L(\theta, a) = (\theta \quad a)^2$. Let the distribution of X be Poisson with parameter $\theta > 0$ and the prior distribution of θ be $G(\alpha, \beta)$. Then the posterior distribution of θ given $X = x$ is

A) $G\left(\alpha x, \frac{\beta}{\beta+1}\right)$

B) $G\left(\alpha + x, \frac{\beta}{\beta+1}\right)$

C) $(\alpha + x, \beta \quad x)$

D) $(\alpha \quad x, \beta \quad x)$

25) Let $\underline{X}_1, \underline{X}_2, \dots, \underline{X}_n$ be n iid (□)random vectors. Then for unbiased estimator ^ of □,

A) n ^ ~ W_p(□, n)

B) (n-1) ^ ~ W_p(□, n)

C) n ^~ W_p(□, n-1)

D) (n-1) ^ ~ W_p(□, n-1)

26) Suppose a cricket ball manufacturing company formed lots of 500 balls. To check the quality of the lots, the buyer draws 20 balls fromeach lot and accepts the lot if the sample contains at the most 1 defective ball. If the quality of submitted lot is 0.03, the AOQ for this planis

A) 0.015

B) 0.025

C) 0.035

D) 0.045

27) The quality characteristic of a manufactured product is monitored by a control chart that is designed to detect the shift with probability $(1 \quad \alpha)$. The expected number of samples analyzed before the shift is detected is

A) $1/\alpha$

B) $1/(1 \quad \alpha)$

C) $1/\alpha^2$

D) $1/(1 \quad \alpha^2)$

28) Suppose that a parallel system is composed of two identical components, each with failure rate = 0.01. The system reliability at t = 10 Hours is

A) 0.69

B) 0.79

C) 0.89

D) 0.99

29) The reliability function of a component is $R(t) = e^{-\lambda t^{\beta}}, \lambda > 0, \beta > 0, t \quad 0$. The component has increasing failure rate (IFR) if

A) $\lambda > 1$

B) $\beta > 1$

C) $\lambda < 1$

D) $\beta < 1$

30) Which of the following six-sigma approaches incorporate tools from the Lean methodology?
A) DMAIC
B) DMADV
C) DMEDI
D) DFSS

31) Which of the following tasks is not involved in the "Improve" phase of DMAIC?
A) Conduct root cause analysis
B) Develop Evaluation Criteria
C) Evaluate for risk
D) Measure results

32) If line segment joining any two points in a set belongs to that set, then such set is called _____ set.
A) bounded
B) concave
C) convex
D) closed

33) If arrival rate is 3 customers/day and service rate is 5 customers/day for an M/M/1queueing system, the expected number of customers in the system at certain day is
A) 1.5
B) 2
C) 3
D) 5

34) For a linear programming problem

$min\ \underline{c}\ \underline{x},$

Subject to $A\underline{x}\quad \underline{b},$

$\underline{x}\quad 0,$

the second variable in the optimal dual solution is positive. Then in theoptimal primal solution

A) the second variable must be zero.
B) the second variable must be positive.
C) slack variable for the second constraint is zero.
D) slack variable for the second constraint is positive.

35) Consider a queue with interarrival time having exponential distribution with mean $1/\lambda$ and service time having exponential distribution with mean $1/\mu$. Which of the following are true?
A) If $0 < \lambda < \mu$, then the queue length has limiting distribution Poisson($\mu\quad\lambda$)
B) If $0 < \mu < \lambda$, then the queue length has limiting distribution Poisson($\lambda\quad\mu$)
C) If $0 < \lambda < \mu$, then the queue length has limiting distribution which is geometric.
D) If $0 < \mu < \lambda$, then the queue length has limiting distribution which is geometric.

36) For an inventory model with quantity discounts, the optimal order quantity depends on
A) the highest unit cost
B) the lowest unit cost
C) the average unit cost
D) None of these

37) A fish stall sells a variety of fish at the rate of Rs 50 per kg on the day of catching. If the stall fails to sell the catch on the same day, it pays for storage at the rate of Rs 3 per kg and the price fetched is Rs 45 per kg on the next day. Past record shows that there is an unlimited demand for one day old fish. It has been found from the past record that daily demand forms a distribution with f (x) = 0.06 –
100. Then the amount of fish that should be procured everyday (in kgs.) so that the total expected cost is minimum will be
A) 78.5 kg
B) 80.6 kg
C) 82.6 kg
D) 86.25 kg

38) Turing test is designed to test _______
A) whether or not a computer is capable of thinking like a human
B) whether or not a computer is capable of thinking rationally
C) whether or not a computer is capable of acting like a human
D) whether or not a computer is capable of acting rationally

39) Which of the following is not an example of introducing inductive bias in linear regression modeling?
A) Assumption of linearity
B) Use of least squares approach
C) Assumption of normality
D) Assuming presence of intercept

40) As we increase the flexibility of machine learning model$\hat{f}$,
A) The bias initially increases faster than the rate at which variance decreases
B) The variance initially increases faster than the rate at which bias decreases
C) The variance initially decreases faster than the rate at which bias increases
D) The bias initially decreases faster than the rate at which variance increases.

41) In machine learning, as the amount of training data increases

A) Training error decreases whereas generalization error increases.

B) Training error increases whereas generalization error decreases.

C) Both training error and generalization error increase.

D) Both training error and generalization error decrease.

42) 'Markov Chain Monte Carlo' is a method for ______

A) Sampling from a Probability distribution

B) Computing Posterior probability

C) Density estimation

D) Bayesian classification

43) In Kernel density estimation, which of the following requirements on Kernel K(z) must be relaxed to obtain an unbiased density estimate.

A) $K(z)$ 0

B) $K(z)dz = 1$

C) $K(z)$ is finite everywhere

D) $K(z)$ 0 $as\ z$ ±

44) Ahidden Markov model_______

A) requires transition probabilities of observable states

B) requires transition probabilities of unobservable states

C) requires transition probabilities of observable as well as unobservable states

D) doesn't require transition probabilities

45) Horvitz-Thompson estimator can be applied to

A) Any random sampling scheme

B) Simple random sampling

C) Systematic Sampling

D) PPS sampling

46) The bias of ratio estimator under simple random sampling is

A) $\frac{Cov(\hat{R}\bar{x})}{\bar{X}}$

B) $\frac{Cov(\hat{R}\bar{x})}{\bar{Y}}$

C) $\frac{Cov(\hat{R}\bar{x})}{\bar{X}}$

D) $\frac{Cov(\hat{R}\bar{x})}{\bar{Y}}$

47) In which of the following cases, a BIBD exists

A) v = 6, b = 12, r = 5, k = 3, = 2

B) v = 6, b = 12, r = 4, k = 2, = 2

C) v = 6, b = 10, r = 5, k = 3, = 2

D) v = 5, b = 10, r = 6, k = 3, = 2

48) While analyzing the data of a Latin square of order v, the error degrees of freedom in the analysis of variance isequal to

A) $(v \quad 1)(v \quad 2)$

B) $v(v \quad 1)$

C) $v^2 \quad 2$

D) $v^2 \quad v \quad 2$

49) Heteroscedasticity is more likely a problem of

A) Cross section data

B) Time series data

C) Pooled data

D) All of above

50) The Goldfeld–Quandt Test is ____

A) a test for detection of multicollinearity

B) a test for detection of heteroscedasticity

C) a test for detection of autocorrelation

D) a test for linear hypothesis in the presence of heteroscedasticity.

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Rough Work: