THE MAHARAJA SAYAJIRAO UNIVERSITY OF BARODA, VADODARA

Ph. D. ENTRANCE TEST (PET) – 27th January 2019

Signature of Invigilators

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Roll. No. | | | | | | |

(in figures as in Hall Ticket)

Roll No. _____

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(in words)

Mathematical Sciences
(19/28)

Maximum Marks: 50 **No. Of Printed Pages : 8**

Instruction for the Candidate:

1. Write your Roll Number in the space provided on the top of this page.
2. This paper consists of **FIFTY (50)** multiple choice type questions. Each Question carries **ONE (1)** mark.
3. At the commencement of examination, the question booklet will be given to you. In the first 5 minutes, you are requested to open the booklet and compulsorily examine it as below:
    a. To have access to the Question Booklet, tear off the paper seal on the edge of this cover page, Do not accept a booklet without sticker seal and do not accept an open booklet.
    b. Tally the number of pages and number of questions in the booklet with the information printed on the cover page. Faculty booklets due to pages/questions missing or duplicate or not in serial order or any other discrepancy should be got replaced immediately by a correct booklet from the invigilator within the period of 5 minutes. Afterwards, neither the Question Booklet will be replaced nor any extra time will be given.
    c. After this verification is over, the Test Booklet Number should be entered on the OMR Answer Sheet and the OMR Answer Sheet Number should be entered on this Test Booklet.
4. Each item has four alternative responses marked (A), (B), (C) and (D). You have to darken the circle as indicated below on the correct response against each item.

Example: Ⓐ ● Ⓒ Ⓓ where (B) is correct response.

5. Your responses to the items are to be indicated on the OMR Answer Sheet under Paper – II only. If you mark your response at any place other than in the circle in the OMR Answer Sheet, it will not be evaluated.
6. Read instructions given inside carefully.
7. Rough Work is to be done in the end of this booklet.
8. If you write your Name, Roll Number, Phone Number or put any mark on any part of the OMR Answer Sheet, except for the space allotted for the relevant entries, which may disclose your identity, or use abusive language or employ any other unfair means, such as change of response by scratching or using white fluid, you will render yourself liable to disqualification.
9. You have to return the original OMR Answer Sheet to the invigilator at the end of the examination compulsorily and must not carry it with you outside the Examination Hall. You are however, allowed to carry original question booklet and duplicate copy of OMR Answer Sheet on conclusion of examination
10. Use only Blue/ Black Ball point pen.
11. Use of any calculator or log table etc., is prohibited.
12. There shall be no negative marking.

Mathematical Sciences

(19/28)

Note: This paper contains **FIFTY (50)** multiple-choice questions. Each Question carries **ONE (1)** mark.

01) The set $\{r \in \mathbb{Q}: r^2 < 5\}$
- A) has no supremum in $\mathbb{R}$
- B) has supremum in $\mathbb{R}$ but no supremum in $\mathbb{Q}$
- C) has supremum in $\mathbb{Q}$
- D) is not bounded above in $\mathbb{R}$.

02) The sequence $\left\{(999)^{1/n}\right\}$
- A) converges to 0
- B) converges to 1
- C) diverges to ∞
- D) converges to 999.

03) Which of the following series are divergent?

(i) $\sum_{n=1}^{\infty}\left((n^3+1)^{\frac{1}{3}}-n\right)$

(ii) $\sum_{n=1}^{\infty}\frac{1}{n^{1+1/n}}$,

(iii) $\sum_{n=1}^{\infty}\sqrt{\frac{4n^3+5}{5n^5+6}}$.

- A) (i), (ii) and (iii)
- B) (i) and (ii)
- C) (ii) and (iii)
- D) (i) and (iii).

04) Which of the following functions is not uniformly continuous on $(0,1)$?
- A) $x\sin\frac{1}{\sqrt{x}}$
- B) x^2
- C) $\frac{\sin x}{x}$
- D) $e^x\cos\frac{1}{x}$.

05) If f is a monotonically decreasing function defined on $[a,b]$, $a<b$,and $E=\{x\in[a,b]: f \text{ is not differentiable at } x\}$, then
- A) $mE = b-a$
- B) $mE = 0$
- C) $0 < mE < b-a$
- D) E is finite.

06) If $[x]$ denotes the greatest integer less than, or equal to, x, then the limit $\lim_{x\to 0}\frac{[x]}{x}$
- A) does not exist
- B) is equal to 1
- C) is equal to 0
- D) is equal to -1.

07) For a non-empty subset S and a point x in a connected metric space (X,d), let $d(x,S)=\inf\{d(x,y): y\in S\}$. Which of the following statements is false?
- A) If S is open and $d(x,S)>0$, then x is not an accumulation point of S.
- B) If S is closed and $d(x,S)>0$, then S does not contain x.
- C) If S is open and $d(x,S)=0$, then $x\in S$.
- D) If S is closed and $d(x,S)>0$, then x is not an accumulation point of S.

08) Let $f:\mathbb{R}^3\to\mathbb{R}^2$ be defined as $f(x,y,z)=(2x+y,3y-4z)$, $(x,y,z)\in\mathbb{R}^3$. Then which of the following is not true?
- A) f is differentiable only at $(0,0,0)$.
- B) $(Df)(x,y,z)=f$ for every $(x,y,z)\in\mathbb{R}^3$.
- C) f is differentiable in $\mathbb{R}^3$.
- D) $(Df)(0,0,0)=f$.

09) The radius of convergence of the power series $\sum_{n=1}^{\infty}\frac{7^n}{n!}z^n$ is
- A) 7
- B) $\frac{1}{7}$
- C) 0
- D) ∞

10) The series $\sum_{n=1}^{\infty}\frac{\cos n\pi}{2n+1}$ is
- A) not alternating
- B) alternating but not convergent
- C) convergent but not absolutely convergent
- D) absolutely convergent.

11) Let $f:[0,1]\to\mathbb{R}$ be defined as $f(0)=0$ and $f(x)=\frac{1}{n}$ if $\frac{1}{n+1}<x\leq\frac{1}{n}$ for $n=1,2,\ldots$. Then on $[0,1]$, f is
- A) not Riemann integrable
- B) Riemann integrable but not continuous
- C) continuous but not monotonic
- D) Riemann integrable as well as continuous.

12) The integral $\int_1^{\infty}\frac{1}{\sqrt{x}}dx$
- A) is equal to $2\sqrt{x}$
- B) is equal to 2
- C) is equal to $\frac{1}{2}$
- D) does not exist.

13) A harmonic conjugate of $u(x,y) = 2x(1-y)$ is
A) $6xy\ -6xy$
B) $3y^2 - 3x^2$
C) $x^2 - y^2 + 2y$
D) $x^3 - 3xy^2$

14) The Laurent series expansion of the function

$f(z) = \frac{e^z}{z(z^2+1)}$ in the domain $0 < |z| < 1$ is

A) $\frac{1}{z^2} + z - \frac{1}{3}z - \frac{5}{6}z^2 + \cdots$
B) $\frac{1}{z} + 1 - \frac{1}{2}z - \frac{5}{6}z^2 + \cdots$
C) $\frac{2}{z} + z - \frac{1}{2}z^2 - \frac{5}{6}z^3 + \cdots$
D) $\frac{3}{z} + 2 - \frac{1}{3}z - \frac{1}{4}z^2 + \cdots$

15) The value of the integral $\int_C \frac{z\,dz}{2z+1}dz$, where C is the positively oriented circle $|z| = 2$ is
A) $\frac{i\pi}{2}$
B) $\frac{\pi}{3}$
C) $\frac{-i\pi}{2}$
D) $\frac{\pi}{4}$

16) The linear fractional transformation that maps the points $z_1 = 2$, $z_2 = i$, $z_3 = -2$on to the points $w_1 = 1$, $w_2 = i$, $w_3 = -1$ is given by
A) $w = \frac{z+2i}{z+4}$
B) $w = \frac{3z-2i}{z+6}$
C) $w = \frac{z+5i}{z+3}$
D) $w = \frac{3z+2i}{iz+6}$

17) The residue at $z = i$ of the function $\frac{z^2-2z}{(z-i)^3}$ is
A) 1
B) i
C) $-i$
D) $2i$

18) Eigen values of a real symmetric matrix are always a
A) positive real number
B) negative real number
C) purely imaginary number
D) real number

19) Let W_1 and W_2 be finite dimensional subspaces of a vector space V, with $\dim W_1 = 6$, $\dim W_2 = 5$ and $\dim(W_1 + W_2) = 9$. Then
A) $W_1 \cap W_2$ need not be a finite dimensional subspace of V
B) there exists a non-zero vector u such that $W_1 \cap W_2 = [u]$
C) there exists vectors $u_1, u_2 \in W_1 \cap W_2$ such that $\{u_1, u_2\}$ is a basis for $W_1 \cap W_2$
D) dimension of $W_1 \cap W_2$ must be between 3 and 6

20) The nullspace of a 3 by 4 matrix A is the span $[(1,1,0,0),(-1,-1,\ 0,0)]$. Then the rank of A is equal to
A) 1
B) 2
C) 3
D) 4

21) Let W be a subspace of a vector space V and let S be a linearly dependent set in W. Which of the following statement is false?
A) If S_1 is a subset of V such that $S \subseteq S_1$ then S_1 must be linearly dependent.
B) If $x \in V$ and $x \notin W$ then$S \cup \{x\}$ is linearly dependent.
C) There exists a subset $S_1 \subset S$ such that S_1 is linearly independent.
D) Every nonempty subset of S is linearly dependent.

22) A cyclic group of order 60 has
A) 16 generators
B) 20 generators
C) 12 generators
D) 32 generators

23) Let R be an integral domain and let $R[x]$ be the polynomial ring in variable x over R. Then which of the following statement is true?
A) $R[x]$ is an Euclidean domain
B) $R[x]$ is an integral domain
C) $R[x]$ is a field
D) $R[x]$ is a division ring

24) Which of the following is not a field?
A) $C[0,1]/I$, where $I = \{f \in C[0,\ 1] \mid f\left(\frac{1}{3}\right) = 0\}$
B) $\mathbb{Z}/2\mathbb{Z}$
C) $\mathbb{Z}_{11}/(x^2+1)$
D) $\mathbb{Z}/10\mathbb{Z}$

25) Let $\mathbb{C}$ be the field of complex numbers and let $\mathbb{C}^*$ be the group of nonzero complex numbers under multiplication. Then which of the following is true?
A) $\mathbb{C}^*$ is cyclic
B) $\mathbb{C}^*$ has finitely many finite subgroups.
C) Every finite subgroup of $\mathbb{C}^*$ is cyclic
D) Every proper subgroup of $\mathbb{C}^*$is cyclic

26) Which one of the following binary operations on set of natural numbers $\mathbb{N}$ is commutative?
A) $\boldsymbol{a * b = a^b}$
B) $\boldsymbol{a * b = 2a + 3b}$
C) $\boldsymbol{a * b = 3a + 2b}$
D) $\boldsymbol{a * b = |a - b| + 1}$

27) Which one of the following matrices is idempotent?
A) $\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$
B) $\begin{pmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}$
C) $\begin{pmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$
D) $\begin{pmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} \end{pmatrix}$

28) Which of the following matrices is not in row-reduced echelon form?
A) $\begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}$
B) $\begin{pmatrix} \mathbf{1} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix}$
C) $\begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$
D) $\begin{pmatrix} \mathbf{1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} \end{pmatrix}$

29) If $\boldsymbol{p(x)}$ is a differentiable function and $\boldsymbol{p(x) > 0}$, then the Wronskian of solutions $\boldsymbol{W(x)}$ of the equation $\frac{\boldsymbol{d}}{\boldsymbol{dx}}\left[\boldsymbol{p(x)}\frac{\boldsymbol{dy}}{\boldsymbol{dx}}\right] + \boldsymbol{q(x)y} = \boldsymbol{0}$ is
A) $\boldsymbol{W(x)} = \frac{\boldsymbol{c}}{\boldsymbol{p(x)}}$, $\boldsymbol{c}$ being a constant
B) $\boldsymbol{W(x)} = \boldsymbol{p(x)}\frac{\boldsymbol{dq(x)}}{\boldsymbol{dx}} + \boldsymbol{q(x)}\frac{\boldsymbol{dp(x)}}{\boldsymbol{dx}}$
C) $\boldsymbol{W(x)} = \boldsymbol{c\, exp}[-\int \boldsymbol{q(x)dx}]$, $\boldsymbol{c}$ being a constant
D) $\boldsymbol{W(x)} = \boldsymbol{0}$.

30) If one solution of the equation $\frac{\boldsymbol{d^2y}}{\boldsymbol{dx^2}} - \boldsymbol{4x}\frac{\boldsymbol{dy}}{\boldsymbol{dx}} + (4x^2 - 2)y = 0$ is $y_1(x) = e^{x^2}$, then the other linearly independent solution $y_2(x)$ is
A) $y_2(x) = e^{-x^2}$
B) $y_2(x) = x^2 e^{x^2}$
C) $y_2(x) = xe^{x^2}$
D) $y_2(x) = e^{x^2} logx$

31) The extremals of the functional $v[y(x)] = \int_{x_0}^{x_1} \frac{1}{x}\sqrt{1 + y'^2}\, dx$ are
A) circles with centres on y-axis
B) circles with centres on x-axis
C) ellipses
D) Parabolas

32) Which of the following statements is not true for the equation $x^2\frac{\partial^2 u}{\partial x^2} - y^2\frac{\partial^2 u}{\partial y^2} - 2y\frac{\partial u}{\partial y} = 0$?
A) Equation is hyperbolic on $\mathbb{R}^2\backslash\{(x, y): x = 0, y = 0\}$ and the variables ξ and η for reducing the equation to canonical form are $\xi = \frac{y}{x}$ and $\eta = xy$.
B) The equation is parabolic on $x = 0,\ y = 0$
C) The equation is not elliptic
D) The equation is hyperbolic on $\mathbb{R}^2$.

33) The complete integral of the equation $z = px + qy + \frac{pq}{pq - p -}$,where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$, represents
A) all planes with sum of intercepts on three coordinate axes unity
B) all planes passing through origin
C) all planes parallel to xy-plane
D) all planes whose distance from origin is unity

34) If two variables, x and y, have a very strong linear relationship, then
A) There is evidence that x causes a change in y
B) There is evidence that y causes a change in x
C) There might not be any causal relationship between x and y
D) None of these alternatives is correct

35) Consider the following statements:
I. The bias of ratio estimator for population mean becomes zero if the regression of Y on X passes through the origin.
II. The ratio estimator of population mean is more efficient than sample mean when the correlation coefficient between X and Y is greater than zero.
Which of the above statements is/are correct?
A) I only
B) II only
C) Both I and II
D) Neither I nor II

36) Let $X_1, X_2, ...X_n$ be iid with $f(x) = \theta x^{\theta-1}, 0 < x < 1, \theta > 0.$ Then the Cramer-Rao Lower Bound for estimating θ is

A) $n\theta$

B) $\frac{\theta}{n}$

C) $\frac{\theta^2}{n}$

D) $\frac{\theta^2}{n^2}$

37) Which of the following statements is not true?

A) If Θ_0 and Θ_1 are both composite, then we need to find a UMP test
B) The class of all test functions of size α is convex
C) UMP test always exists for every problem $(\alpha, \Theta_0, \Theta_1)$
D) All of these.

38) Let Y_1, Y_2, Y_3 and Y_4 be uncorrelated observations with common unknown variance σ^2 and expectations given by

$E(Y_1) = \beta_1 + \beta_2 + \beta_3 = E(Y_2)$, $E(Y_3) = \beta_1 - \beta_2 = E(Y_4)$,

where β_1, β_2 and β_3 are unknown parameters. Define $e_1 = (Y_1 - Y_2)/\sqrt{2}$ and $e_2 = (Y_3 - Y_4)/\sqrt{2}$. An unbiased estimator of σ^2 is

A) $(e_1^2 - e_2^2)/2$
B) $(e_1^2 + e_2^2)/2$
C) $(e_1^2 + e_2^2)/4$
D) $e_1^2 + e_2^2$

39) Let X be a random variable having the probability function: $f(x, \theta) = \binom{n}{x}\theta^x(1-\theta)^{n-x}, x = 0,1,2,...,n.$ If $d(x) = \frac{x}{n}$, then the risk function $R(\theta, d)$ under squared error loss function is:

A) $\frac{\theta(\theta-1)}{n}$

B) $\frac{\theta(\theta+1)}{n}$

C) $\frac{\theta(1-\theta)}{n}$

D) $\frac{\theta^2}{n}$

40) Let x_1, x_2 be iid as exponential with density: $f(x) = \frac{1}{\theta}e^{-x/\theta}, x \geq 0, \theta > 0$. Let $u_1 = 0.6x_1 + 0.4x_2$ and $u_2 = x_1 + x_2$. Which of the u_1 and u_2 is sufficient for θ?

A) Only u_1
B) Only u_2
C) Both u_1 and u_2
D) Neither u_1 and u_2

41) Suppose for a normal distribution the population mean is 50 and the population variance is 36. The distribution of the sample mean of 100 observations has which of the following normal distribution?

A) N(50, 36)
B) N(50, 3.6)
C) N(50, 0.36)
D) N(50, 0.6)

42) Consider the following five observations on (X, Y): (0, 1), (1, 2), (2, 3), (3, 2), (4, 1). Then

A) The least square liner regression of Y on X is Y = 9/5
B) The least square linear regression of X and Y is X = 2Y + 3
C) The correlation coefficient between X and Y is +1
D) None of the above.

43) If 3, 8, 5, 4 and 10 are exponential samples with mean θ. The Fisher information function evaluated at $\theta=2$ is
A) 0.50
B) 0.80
C) 1.20
D) 1.25

44) Which of the following is TRUE with regard to multicollinearity in multiple regression analysis?
A) It increases the influence of the individual variables in the model
B) It reduces the predictive power of the model
C) It increases the reliability of the regression coefficients
D) It reduces the effectiveness of sensitivity analysis of the model

45) With the usual notations, find p for a binomial random variable x, if n= 6 and if 9 P (X=1) = P(X=2).
A) 12/15
B) 18/23
C) 19/27
D) 9/14

46) If the pgf of a certain distribution is given as P(s) = $3s^2+2s+6$. What is the mean of this distribution?
A) 4
B) 6
C) 8
D) 10

47) The means of two large samples of sizes 1000 and 2000 are 67.5 and 68.0 respectively. The standard error of the difference of the mean when the two populations have equal standard deviation 2.5 is
A) 0.210
B) 0.193
C) 0.097
D) 0.002

48) According to Mendelian inheritance, offspring of a certain crossing should be colored red, black, or white in the ratios 9:3:4. If an experiment gave 72, 40, and 48 offspring in those categories in an observed sample, what should be the expected number of black colored offsprings if we assume that the Mendelian theory is substantiated?
A) 25
B) 30
C) 35
D) 38

49) Consider the following statements:
I. Correlation is a measure of degree of association between two variables
II. Correlation measures linearity of variables
III. Correlationis a non-directionalquantity
Which of the above statements are correct?
A) Only I is correct
B) Both I and II are correct
C) Both I and III are correct
D) Both II and III are correct

50) Let N(t) be a Poisson process with constant intensity function on R. What is the covariance of N(s) and N(t)?
A) λs, if s<t
B) λ(t-s), if s<t
C) λ(s-t), if (t<s)
D) λ(s+t)

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Rough Work: